

Thermodynamic Cycle Analysis of Magnetohydrodynamic-Bypass Airbreathing Hypersonic Engines

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Introduction

Established analyses of conventional ramjet/scramjet performance characteristics indicate that a considerable decrease in efficiency can be expected at off-design flight conditions. This can be explained, in large part, by the deterioration of intake mass flow and limited inlet compression at low flight speeds and by the onset of thrust degradation effects associated with increased burner entry temperature at high flight speeds. In combination, these effects tend to impose lower and upper Mach number limits for practical flight. It has been noted, however, that Magnetohydrodynamic (MHD) energy management techniques represent a possible means for extending the flight Mach number envelope of conventional engines.^[1-3] By transferring enthalpy between different stages of the engine cycle, it appears that the onset of thrust degradation may be delayed to higher flight speeds. Obviously, the introduction of additional process inefficiencies is inevitable with this approach, but it is believed that these losses are more than compensated through optimization of the combustion process.

The fundamental idea is to use MHD energy conversion processes to extract and bypass a portion of the intake kinetic energy around the burner. We refer to this general class of propulsion system as an MHD-bypass engine. In its generic configuration, an MHD generator is placed between the inlet and the burner as a means of converting flow kinetic energy into electrical power. In this way, the overall static temperature rise associated with inlet flow deceleration can be actively constrained, and the propulsion system is able to reach a higher freestream Mach number (i.e. higher inlet stagnation enthalpy) before the burner entry temperature exceeds the design limit. Furthermore, given a fixed limit for burner entry temperature, the inlet MHD generator can decelerate the flow to a lower velocity than that attainable using a simple adiabatic compression process. Thus, it is possible to increase the freestream Mach number for which the flow remains subsonic throughout the burner, and the ramjet mode can

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be sustained to much higher flight Mach numbers. In this scheme, it is necessary to divert a fraction of the bypassed power for operation of a pre-ionizer in the diffuser. Non-equilibrium ionization of the inlet air is essential for attaining the electrical conductivity levels needed for effective MHD interaction in the generator. Excess electrical power is reintroduced to the cycle as an increase in total enthalpy using an MHD accelerator that is located between the burner and nozzle.

Although the MHD-bypass engine concept is of considerable contemporary interest to the hypersonics community, a review of the open literature has revealed only a small number of analytical investigations aimed at performance prediction of these systems.^[4-7] These analyses have demonstrated favorable performance characteristics under highly specific flight conditions, but a simplified thermodynamic cycle analysis would also be useful for the purpose of assessing systems performance potential on a more generalized basis.

In this paper, we quantitatively assess the performance potential and scientific feasibility of MHD-bypass airbreathing hypersonic engines using ideal gasdynamics and fundamental thermodynamic principles. The cycle analysis, based on a thermally and calorically perfect gas, incorporates strategically placed magnetohydrodynamic devices in the flow path and accounts for aerodynamic losses and thermodynamic process efficiencies in the various engine components. MHD device performance is completely described in terms of an enthalpy extraction/addition parameter and an isentropic efficiency. A provision is also made for diverting a fraction of the bypassed electrical power to an air pre-ionizer located at the inlet of the diffuser. The power consumed by the pre-ionizer is considered only as a variable parameter. The detailed ionization kinetics are not addressed in this paper. The analysis, while certainly not applicable to detailed performance analysis, is fundamental and is suitable as an indicator of potential performance gains associated with MHD energy bypass. It also reveals the flight Mach number range over which the system can effectively operate and suggests the range of component efficiencies needed for successful implementation.

The prime technical objective of this work is to find simple closed form solutions for the performance of MHD-bypass airbreathing engines and use them to expose important trends and sensitivities, as well as to establish thermodynamic feasibility, without recourse to elaborate thermochemical calculations. From this standpoint, our methodology adheres to the analysis philosophy and spirit of Builder's pioneering approach.^[8]

Performance Model

Thermodynamic Cycle

We consider the thermodynamic cycle of an airbreathing hypersonic engine of the ramjet/scramjet class that is augmented with an MHD energy management system. The geometrical configuration investigated here follows the generic MHD-bypass engine concept. The various engine components are shown in Fig. 1 along with selected reference stations at critical axial positions along the engine flowpath. This figure also includes the entropy-

enthalpy diagram for a modified Brayton cycle as represented by a sequence of eight process trajectories. These process trajectories may be summarized as follows: (a→1) air pre-ionization and heat addition; (1→2) adiabatic compression and flow deceleration; (2→3) MHD conversion of total enthalpy to electrical power and deceleration of the flow; (3→4) constant static pressure and frictionless heat addition; (4→5) adiabatic expansion to prevent the temperature from exceeding design limits in the generator; (5→6) MHD conversion of electrical power to total flow enthalpy and acceleration of the flow; and, (6→10) adiabatic expansion and acceleration of the exhaust flow.

Thermal Design Constraints

Presently, materials used to construct the walls of combustion chambers and nozzles cannot tolerate temperatures much above 1200 K. Unlike gas turbine engines, however, the surfaces of ramjet/scramjet combustors can be kept much cooler than the main fluid stream by providing a shielding layer of relatively cool air near the wall. Therefore, this class of engine can accept higher peak burner temperatures and can operate at higher flight Mach numbers. In this way, the relative performance and operating range of ramjet/scramjet engines is greatly improved and extended over the gas turbine engine. As the flight Mach number continues to increase, however, the combustor inlet temperature also increases until, at some limiting Mach number, the peak cycle temperature begins to approach the temperature limit set by the wall materials and cooling methods. Because the heat energy added by fuel combustion can generate unacceptable temperature levels at the burner exit, we introduce a temperature design constraint on the peak combustor stagnation temperature in the form:

$$T_{0,4} \leq T_{0,\text{lim}} \quad (1)$$

Because the stagnation temperature is always increased in the MHD accelerator and because this device will be subject to similar thermal design limits as the burner, it is possible to introduce an additional design constraint on the peak accelerator temperature in the form:

$$T_{0,6} \leq T_{0,\text{lim}} \quad (2)$$

Even if material temperature limits could be extended, the static temperature at the burner entrance cannot be increased indefinitely. At temperatures approaching 2000 K, energy losses due to unequilibrated dissociation begin to impair performance and eventually overwhelm any benefits associated with increased cycle temperature. Based on the well known thermodynamic characteristics of air, the maximum allowable burner entry temperature is normally found to fall in the range 1400 - 1700 K.^[9] Therefore, a typical value for design is near 1600 K. For analysis purposes, we impose this practical thermodynamic constraint on design by requiring that the static temperature at the burner entrance not exceed a specified limiting value:

$$T_3 \leq T_{3,\text{lim}} \quad (3)$$

This limiting value should be low enough that air may be approximated as a thermally perfect gas with no dissociation effects during the entire inlet conditioning process.

Thermodynamic Analysis of Engine Processes

Our approach to estimating the performance of an MHD-bypass hypersonic engine is to perform a thermodynamic analysis of the appropriately modified Brayton cycle. To carry out this analysis, we assume that the working medium can be treated as a pure substance throughout the engine flowpath and that the working medium is thermally and calorically perfect. Because practical thermodynamic and aerodynamic losses can lead to significant entropy increases and stagnation pressure changes along the flow path, it is necessary to introduce various process efficiencies into the analysis. These efficiencies are introduced in terms of a stagnation pressure ratio ($\pi = p_{0,\text{exit}}/p_{0,\text{in}}$) for each engine process. Under these assumptions, it is possible to develop relations describing the thermodynamic process trajectories in each component of the engine and to reach a simple closed form solution for specific impulse (I_{sp}) and thrust specific fuel consumption (TSFC).

We begin the development by specifying the free stream total temperature and total pressure in the stream tube of the engine inlet. These properties can be defined in terms of static conditions, which are altitude dependent, and the flight Mach number M_a using the following well known gasdynamic relationships:

$$T_{0,a} = T_a \left(1 + \frac{\gamma - 1}{2} M_a^2 \right) \quad (4)$$

$$p_{0,a} = p_a \left(1 + \frac{\gamma - 1}{2} M_a^2 \right)^{\frac{\gamma}{\gamma - 1}} \quad (5)$$

where γ is the specific heat ratio. The objective at this point is to track the variation in these stagnation properties through the various engine components using the first and second laws of thermodynamics.

Before proceeding with the analysis, it is useful to note that any MHD device can be described in terms of two fundamental thermodynamic parameters: (1) an enthalpy extraction/addition ratio η_N and (2) an isentropic efficiency η_s which accounts for aerodynamic losses as well as Joule dissipation effects. The enthalpy extraction/addition ratio is always defined as the ratio of the stagnation enthalpy change in the device to the inlet stagnation enthalpy:

$$\eta_N = \frac{\Delta h_0}{h_{0,\text{in}}} \quad (6)$$

The isentropic efficiency represents the degree to which the actual process approaches an isentropic process. For a generator (energy extraction), it is defined as the ratio of the actual change in stagnation enthalpy to the ideal (isentropic) change in stagnation enthalpy that would accompany the same change in pressure:

$$\eta_{s(g)} = \frac{\Delta h_{0,\text{actual}}}{\Delta h_{0,\text{isentropic}}} \quad (7)$$

For an accelerator (energy insertion), it is defined as the ratio of the ideal to the actual:

$$\eta_{r(a)} = \frac{\Delta h_{0, \text{isentropic}}}{\Delta h_{0, \text{actual}}} \quad (8)$$

First-Law Analysis

The pre-ionizer takes a fraction χ of the energy produced by the MHD generator and introduces it to the inlet flow through an external energy addition process. The purpose is to increase the ionization level and electrical conductivity of the air before it enters the MHD generator. In the present formulation, we assume that no compression occurs in the pre-ionization region. Application of the first law of thermodynamics to the inlet stream tube in the pre-ionization region yields an energy balance in the form:

$$\dot{m}_a h_{0,1} = \dot{m}_a (h_{0,a} + \xi \chi \eta_{N(g)} h_{0,2}) \quad (9)$$

where h_0 is the specific stagnation enthalpy, \dot{m}_a is the mass flow rate of air through the engine, and $\eta_{N(g)}$ is the enthalpy extraction ratio for the MHD generator, the source of power. The quantity $\eta_{N(g)} h_{0,2}$ is the specific electrical energy extracted by the MHD generator. Using the fact that the flow is adiabatic in the diffuser, $h_{0,2} = h_{0,1}$, we obtain the form:

$$\frac{T_{0,1}}{T_{0,a}} = \frac{h_{0,1}}{h_{0,a}} = \frac{1}{1 - \xi \chi \eta_{N(g)}} \quad (10)$$

Because some degree of nonequilibrium air ionization may be feasible, we have introduced the phenomenological parameter ξ as a measure of the pre-ionization energy which goes into heating of the heavy species. That is, $0 < \xi < 1$ corresponding to nonequilibrium conditions of various degrees.

The compression process in the diffuser is assumed to be adiabatic such that the stagnation temperature remains invariant:

$$T_{0,2} = T_{0,1} = \frac{T_{0,a}}{1 - \xi \chi \eta_{N(g)}} \quad (11)$$

By extracting flow enthalpy and converting it to electrical power, the MHD generator acts as an inlet flow conditioner for the engine. In the MHD generator both total enthalpy and total pressure are decreased. Application of the first law of thermodynamics to the MHD generator yields an energy balance in the form:

$$\dot{m}_a h_{0,3} = \dot{m}_a (h_{0,2} - \eta_{N(g)} h_{0,2}) \quad (12)$$

where $\eta_{N(g)} h_{0,2}$ is the total enthalpy converted to electrical power per unit mass flow of air through the generator. In terms of stagnation temperatures, Eq. (12) can be written in the form:

$$T_{0,3} = T_{0,2} (1 - \eta_{N(g)}) = T_{0,a} \frac{1 - \eta_{N(g)}}{1 - \xi \chi \eta_{N(g)}} \quad (13)$$

where we have introduced Eq. (11) to obtain a result in terms of the free stream stagnation temperature.

In the burner, the stagnation temperature is increased as the chemical energy of the fuel is released as combustion heat. This process is assumed to occur at constant static pressure according to the assumed thermodynamic cycle although actual implementation might correspond more closely to burning in a constant area duct. The energy equation applied to this idealized combustion process, neglecting the enthalpy of the incoming fuel, is

$$\begin{aligned} (\dot{m}_a + \dot{m}_f)h_{0,4} &= \dot{m}_a h_{0,3} + \eta_c \dot{m}_f q_f \\ (1+f)h_{0,4} &= h_{0,3} + \eta_c f q_f \\ (1+f)C_p T_{0,4} &= C_p T_{0,3} + \eta_c f q_f \end{aligned} \quad (14)$$

where \dot{m}_f represents the mass flow rate of fuel, q_f is the fuel heating value, η_c is the combustion efficiency of the burner, C_p is the constant pressure specific heat of the working fluid, and f is the fuel-to-air mass flow ratio. The combustion efficiency is included to account for the fact that some of the chemical energy of the fuel may not be released due to inadequate mixing or reaction time. The parameter may also be used to reflect energy losses associated with chemical dissociation. For constant specific heat, Eq. (14) can be solved for the stagnation temperature at the burner exit

$$T_{0,4} = \frac{T_{0,3} + (\eta_c f q_f / C_p)}{1+f} \leq T_{0,lim} \quad (15)$$

Alternatively, Eq. (14) may be solved for f in the form:

$$f = \frac{(T_{0,4}/T_{0,3}) - 1}{(\eta_c q_f / C_p T_{0,3}) - T_{0,4}/T_{0,3}} \quad (16)$$

where $T_{0,3}$ can be obtained from Eq. (13). It should be apparent that the inequality of Eq. (15) can be satisfied by (1) limiting the value of $T_{0,3}$ through the extraction of enthalpy in the MHD generator and/or (2) constraining the fuel-to-air ratio f such that

$$f \leq \frac{1 - (T_{0,3}/T_{0,lim})}{(\eta_c q_f / C_p T_{0,lim}) - 1} \quad (17)$$

A weak expansion component may be included after the burner to expand and cool the flow enough such that, after passage through the MHD accelerator, the static temperature remains below specified design limits. We assume that the expansion is adiabatic such that the stagnation temperature remains constant ($T_{0,5} = T_{0,4}$).

The MHD accelerator provides a mechanism for enthalpy addition in which the kinetic energy extracted from the inlet is recovered as an augmentation to overall engine thrust. In the accelerator, both total enthalpy and total pressure are increased. Because the generator and accelerator are coupled, the electrical power available for acceleration is limited to the total enthalpy extracted in the inlet minus that diverted for pre-ionization.

Application of the first law of thermodynamics to the MHD accelerator yields an energy balance in the form:

$$\begin{aligned} (\dot{m}_a + \dot{m}_f) h_{0,6} &= (\dot{m}_a + \dot{m}_f) [h_{0,5} + \eta_{N(a)} h_{0,5}] \\ T_{0,6} &= T_{0,5} (1 + \eta_{N(a)}) \end{aligned} \quad (18)$$

where we have introduced the enthalpy addition ratio, $\eta_{N(a)}$. The product $\eta_{N(a)} h_{0,5}$ is the total enthalpy added to the flow per unit mass flow rate of combustion gases through the accelerator.

Because the electrical energy available for acceleration is equal to the total power produced by the generator minus that diverted for pre-ionization, the enthalpy addition ratio of the accelerator is directly related to the enthalpy extraction ratio of the generator according to the following power balance:

$$\eta_{N(a)} (\dot{m}_a + \dot{m}_f) h_{0,5} = (1 - \chi) \eta_{N(g)} \dot{m}_a h_{0,2} \quad (19)$$

or

$$\eta_{N(a)} T_{0,5} = \left(\frac{1}{1+f} \right) (1 - \chi) \eta_{N(g)} T_{0,1} \quad (20)$$

where we have used the fact that $T_{0,2} = T_{0,1}$. Upon substitution of Eq. (20) into Eq. (18) we eliminate $\eta_{N(a)}$ to obtain

$$T_{0,6} = T_{0,5} + \left(\frac{1}{1+f} \right) (1 - \chi) \eta_{N(g)} T_{0,1} \quad (21)$$

Eliminating $T_{0,1}$ using Eq. (10) and noting that $T_{0,5} = T_{0,4}$ we obtain the relation

$$T_{0,6} = T_{0,4} + \left(\frac{1}{1+f} \right) \frac{(1 - \chi) \eta_{N(g)} T_{0,4}}{1 - \xi \chi \eta_{N(g)}} \quad (22)$$

The expansion process in the nozzle is assumed to be adiabatic such that the stagnation temperature remains invariant ($T_{0,10} = T_{0,6}$).

Stagnation Pressure

As the propellant flows through the engine, irreversibilities result in stagnation pressure losses and an increase in entropy. Therefore, real aerodynamic losses (or gains) may be accounted for through the introduction of a stagnation pressure ratio ($\pi = p_{0,\text{exit}}/p_{0,\text{in}}$) for each engine process. Expressions for these π parameters can be developed from fundamental thermodynamic and gasdynamic principles as demonstrated for conventional engine components by Heiser and Pratt^[9] and the reader is referred to this text for detailed derivations. The π parameters for the MHD devices in the engine flow path introduce a novel consideration, however, and we shall briefly develop the appropriate expressions for the generator and accelerator components in terms of their thermodynamic process parameters.

Consider, first, the MHD generator. To develop an expression for the stagnation pressure loss, we begin with the Gibbs equation for an ideal (isentropic) process as applied to stagnation properties

$$\frac{dp_0}{p_0} = \frac{\gamma}{\gamma-1} \frac{dh_0}{h_0} \quad (23)$$

Integrating Eq. (23) through the MHD generator yields the stagnation pressure ratio π_g in terms of the total enthalpy ratio

$$\pi_g = \frac{p_{0,3}}{p_{0,2}} = \left(\frac{h_{0,3}}{h_{0,2}} \right)^{\frac{1}{\gamma-1}} \quad (24)$$

Utilizing the definition of $\eta_{N(g)}$ given by Eq. (6) and the definition of $\eta_{s(g)}$ given by Eq. (7), we may form the following relationship between generator performance parameters:

$$\frac{\eta_{N(g)}}{\eta_{s(g)}} = \frac{(h_{0,2} - h_{0,3})_{isentropic}}{(h_{0,2} - h_{0,3})_{actual}} \frac{(h_{0,2} - h_{0,3})_{actual}}{h_{0,2}} = \left(\frac{h_{0,2} - h_{0,3}}{h_{0,2}} \right)_{isentropic} = 1 - \left(\frac{h_{0,3}}{h_{0,2}} \right)_{isentropic} \quad (25)$$

Upon substituting Eq. (25) into Eq. (24), we obtain the desired expression for the stagnation pressure ratio in the MHD generator

$$\pi_g = \frac{p_{0,3}}{p_{0,2}} = \left(1 - \frac{\eta_{N(g)}}{\eta_{s(g)}} \right)^{\frac{1}{\gamma-1}} \quad (26)$$

Consider, now, the MHD accelerator in which the stagnation pressure must increase. We begin with the Gibbs equation for an ideal (isentropic) process as applied to stagnation properties and integrate through the accelerator to obtain the stagnation pressure ratio in terms of the total enthalpy ratio

$$\pi_a = \frac{p_{0,6}}{p_{0,5}} = \left(\frac{h_{0,6}}{h_{0,5}} \right)^{\frac{1}{\gamma-1}} \quad (27)$$

Utilizing the definition of $\eta_{N(a)}$ given by Eq. (6) and the definition of $\eta_{s(a)}$ given by Eq. (8), we may form the following relationship between accelerator performance parameters:

$$\eta_{N(a)} \eta_{s(a)} = \frac{(h_{0,6} - h_{0,5})_{isentropic}}{(h_{0,6} - h_{0,5})_{actual}} \frac{(h_{0,6} - h_{0,5})_{actual}}{h_{0,5}} = \left(\frac{h_{0,6} - h_{0,5}}{h_{0,5}} \right)_{isentropic} = \left(\frac{h_{0,6}}{h_{0,5}} \right)_{isentropic} - 1 \quad (28)$$

Upon substituting Eq. (28) into Eq. (27), we obtain an expression for the stagnation pressure ratio in the MHD accelerator in terms of the thermodynamic parameters of the device

$$\pi_a = \frac{p_{0,6}}{p_{0,5}} = \left(1 + \eta_{N(a)} \eta_{s(a)} \right)^{\frac{1}{\gamma-1}} \quad (29)$$

If we eliminate $\eta_{N(a)}$ using Eq. (20) and eliminate $T_{0,1}$ using Eq. (10), we obtain the desired final expression for the stagnation pressure ratio in the MHD accelerator

$$\pi_a = \frac{p_{0,6}}{p_{0,5}} = \left[1 + \eta_{s(a)} \left(\frac{1}{1+f} \right) \frac{(1-\chi) \eta_{N(g)} T_{0,a}}{1 - \xi \chi \eta_{N(g)} T_{0,4}} \right]^{\frac{r}{r-1}} \quad (30)$$

where we have also used the fact that $T_{0,5} = T_{0,4}$.

Note that the enthalpy extraction ratio and isentropic efficiency performance parameters fully define the operational characteristics of the generator and accelerator, including Joule dissipation losses. Representative values can be specified based on historical data from past MHD research.

Thrust Analysis

We now have sufficient information to compute the total engine thrust including the effect of aerodynamic losses. Although the π parameters are not truly constant over large Mach number variations and γ can vary significantly through the engine flowpath, the following development will demonstrate the essential performance characteristics of hypersonic airbreathing engines.

First, we utilize the Mach number relation between stagnation and static pressures as defined by Eq. (5) and evaluate these expressions at the inlet and exit of the engine. We then form the ratio between these expressions and solve for the exit Mach number M_e^2 ²

$$M_e^2 = \frac{2}{\gamma-1} \left[\left(1 + \frac{\gamma-1}{2} M_a^2 \right) \left(\frac{p_{0,e}}{p_{0,a}} \frac{p_a}{p_e} \right)^{(r-1)/r} - 1 \right] \quad (31)$$

or

$$M_e^2 = \frac{2}{\gamma-1} \left[\left(1 + \frac{\gamma-1}{2} M_a^2 \right) \left(\frac{p_{0,1}}{p_{0,a}} \frac{p_{0,2}}{p_{0,1}} \frac{p_{0,3}}{p_{0,2}} \frac{p_{0,4}}{p_{0,3}} \frac{p_{0,5}}{p_{0,4}} \frac{p_{0,6}}{p_{0,5}} \frac{p_{0,10}}{p_{0,6}} \frac{p_a}{p_e} \right)^{(r-1)/r} - 1 \right] \quad (32)$$

Thus, in terms of the component stagnation pressure ratios,

$$M_e^2 = \frac{2}{\gamma-1} \left[\left(1 + \frac{\gamma-1}{2} M_a^2 \right) \left(\pi_p \pi_d \pi_g \pi_b \pi_e \pi_a \pi_n \frac{p_a}{p_e} \right)^{(r-1)/r} - 1 \right] \quad (33)$$

where π_p , π_d , π_g , π_b , π_e , π_a , and π_n are the π parameters for the pre-ionizer, diffuser, generator, burner, cooling expansion, accelerator, and nozzle, respectively. We now define a global stagnation pressure ratio parameter Π as

$$\Pi = \left(1 + \frac{\gamma-1}{2} M_a^2 \right) \left(\pi_p \pi_d \pi_g \pi_b \pi_e \pi_a \pi_n \frac{p_a}{p_e} \right)^{(r-1)/r} \quad (34)$$

such that

$$M_e^2 = \frac{2}{\gamma-1} [\Pi - 1] \quad (35)$$

Assuming that heat transfer from the engine is negligible, then the exhaust velocity is given by $u_e = M_e \sqrt{\gamma R T_e}$ or, in terms of the exhaust stagnation temperature

$$u_e = M_e \sqrt{\gamma RT_{0,6} / \left(1 + \frac{\gamma-1}{2} M_e^2 \right)} \quad (36)$$

where we have made use of the fact that $T_{0,10} = T_{0,6}$. The fuel-air ratio necessary to produce the desired value for $T_{0,4}$ is given by Eq. (16).

The thrust for an airbreathing hypersonic engine is defined by

$$F = (\dot{m}_a + \dot{m}_e) u_e - \dot{m}_a u_a + (p_e - p_a) A_e \quad (37)$$

and the thrust per unit mass flow rate of air becomes

$$\frac{F}{\dot{m}_a} = [(1+f) u_e - u_a] + \frac{1}{\dot{m}_a} (p_e - p_a) A_e \quad (38)$$

Substituting Eq. (36) into Eq. (38) yields the desired form

$$\frac{F}{\dot{m}_a} = \left[(1+f) \sqrt{\frac{2\gamma RT_{0,6} (\Pi-1)}{(\gamma-1)\Pi}} - M_a \sqrt{\gamma RT_a} \right] + \frac{p_e A_e}{\dot{m}_a} \left(1 - \frac{p_a}{p_e} \right) \quad (39)$$

where $T_{0,6}$ is obtained from Eq. (22) using $T_{0,4}$ (i.e., $T_{0,\text{lim}}$) as a parameter. The fuel specific impulse may also be determined as

$$I_{sp} = \frac{F}{\dot{w}_f} = \frac{F/\dot{w}_a}{f} \quad (40)$$

where \dot{w}_a and \dot{w}_f are the weight flow rates of air and fuel, respectively. Following conventional practice, the thrust specific fuel consumption (TSFC) is defined as the ratio of fuel mass flow rate to engine thrust

$$TSFC = \frac{\dot{m}_f}{F} = \frac{f}{F/\dot{m}_a} \quad (41)$$

Maximum Flight Mach Number for Subsonic Combustion

The limitation on burner entry temperature leads directly to a restriction on the burner entry Mach number M_3 . That is, the flow can be decelerated only a limited amount before the static temperature at the burner inlet becomes too great for effective combustion, and at some particular flight Mach number, it becomes necessary to transition to a supersonic combustion mode ($M_3 > 1$). For the MHD-bypass engine configuration, however, the burner entry temperature is affected by the enthalpy extraction process in the MHD generator. Indeed, the bypassing of flow enthalpy around the burner enables the vehicle to obtain a higher flight Mach number while maintaining a subsonic combustion mode.

We can easily develop a relation for the burner entry Mach number as a function of the flight Mach number, the enthalpy extraction ratio, and the static temperature limit. We begin with the Mach number relation between stagnation temperature and static temperature at the burner entrance

$$T_{0,3} = T_3 \left(1 + \frac{\gamma-1}{2} M_3^2 \right) \quad (42)$$

Then we eliminate $T_{0,3}$ using Eq. (13) while enforcing a burner entry static temperature limit ($T_{3,\text{lim}}$) and find that the burner entry Mach number is given by

$$M_3 = \sqrt{\frac{2}{\gamma-1} \left[\frac{T_a}{T_{3,\text{lim}}} \frac{1-\eta_g}{1-\chi\eta_g} \left(1 + \frac{\gamma-1}{2} M_a^2 \right) - 1 \right]} \quad (43)$$

This relation implies definite operating restrictions. For example, when

$$M_a < \sqrt{\frac{2}{\gamma-1} \left(\frac{T_{3,\text{lim}}}{T_a} \frac{1-\chi\eta_g}{1-\eta_g} - 1 \right)} \quad (44)$$

no solution is physically possible since $T_{3,\text{lim}}$ would exceed the stagnation temperature of the freestream flow. Furthermore, when

$$M_a > \sqrt{\frac{2}{\gamma-1} \left[\left(\frac{\gamma+1}{2} \right) \frac{T_{3,\text{lim}}}{T_a} \frac{1-\chi\eta_g}{1-\eta_g} - 1 \right]} \quad (45)$$

the flow entering the burner must remain supersonic or $T_{3,\text{lim}}$ will be exceeded.

Performance Evaluation

It is instructive to compare the performance of an MHD-bypass hypersonic airbreathing engine with that of a conventional ramjet system. For this purpose, Hill and Peterson's well known analysis for a nonideal ramjet represents a suitable baseline case.^[10] Indeed, their analytical results are directly recoverable from our development by simply eliminating all MHD interaction (i.e., setting $\eta_{N(g)} = \eta_{N(a)} = 0$). Following Hill and Peterson, we assumed a freestream temperature of $T_a = 220$ K and used the thermodynamic properties of air throughout the engine flow path. The fuel heat of combustion was taken to be $q_f = 45 \times 10^3$ kJ/kg with 100% combustion efficiency and a stagnation temperature limit was enforced at the burner exit. As in Hill and Peterson's case, we considered constant stagnation pressure ratios of $\pi_d = 0.7$, $\pi_b = 0.95$, and $\pi_n = 0.98$ for the diffuser, burner, and nozzle, respectively. As previously noted, the calculations are limited by the fact that γ is not constant throughout the engine flowpath and by the fact that the stagnation pressure ratios are not independent of flight Mach number. These coefficients were taken as constants solely for the purpose of maintaining simplicity and clarity. We have optimistically assumed that $\eta_{s(g)} = \eta_{s(a)} = 0.9$ and computed π_g (loss) and π_a (gain) using Eqs. (26) and (30), respectively. The power needed to drive the pre-ionizer is difficult to estimate at this juncture and we utilized a value of $\chi = 0.05$ with $\xi = 1$ for demonstration purposes.

The computed specific thrust is shown as a function of flight Mach number in Fig. 2 for enthalpy extraction ratios of $\eta_{N(g)} = 0, 0.25$, and 0.5 and a maximum burner stagnation temperature of 3000 K. The case without any

MHD interaction corresponds directly with Hill and Peterson's results and is demonstrative of a conventional airbreathing hypersonic engine. The specific thrust in this instance peaks at a flight Mach number between 2 and 3 after which it steadily decreases to zero near $M_a = 8$. The results with MHD interaction demonstrate a reduction in peak performance and a shifting of the specific thrust curve to higher flight Mach numbers. The loss in performance using MHD-bypass is due to increased cycle inefficiencies associated with the MHD energy conversion devices. However, the ability to bypass flow enthalpy around the burner permits optimization of the combustion process at higher flight Mach numbers and enables the production of thrust in a flight regime not obtainable with conventional engines. This capability is the primary advantage of the MHD-bypass concept.

The combustion optimization attributes can be better appreciated, perhaps, through an inspection of the maximum flight Mach number which can be achieved with MHD-bypass of flow enthalpy while maintaining subsonic combustion conditions. This effect, shown in Fig. 3, is demonstrated through solution of Eq. (45) assuming a maximum static burner entry temperature of $T_{3,lim} = 1600$ K.

It has not escaped the authors' attention that the present cycle analysis is greatly limited by the assumption of constant stagnation pressure ratios for the various engine components. Other critical system level issues include a suitable pre-ionizer for nonequilibrium ionization of air, flight-weight magnets, optimization of generator/accelerator designs, power conditioning systems, and intelligent controls. Nevertheless, the analysis does reveal the essential operational benefits of MHD-bypass engines and demonstrates fundamental thermodynamic feasibility. We conclude that such systems hold significant promise for extending the effective operating range of conventional airbreathing hypersonic engines and believe that the present results justify a more in-depth analysis in which the stagnation pressure ratios become dependent on flight Mach number and the combustion process is modeled more accurately.

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Figure Captions

Figure 1: Generalized configuration of an airbreathing hypersonic engine with MHD energy management system. Reference stations and process terminology are indicated. Also shown is the enthalpy-entropy cycle diagram for the generic MHD-bypass airbreathing hypersonic engine configuration.

Figure 2: Specific thrust characteristics as a function of flight Mach number for $\eta_{N(g)} = 0, 0.25$, and 0.5 assuming $\eta_{s(g)} = \eta_{s(a)} = 0.9$ and $T_{0,lim} = 3000$ K.

Figure 3: Maximum flight Mach number for maintaining subsonic combustion conditions as a function of the generator enthalpy extraction ratio. Assumes a maximum static burner entry temperature of $T_{3,lim} = 1600$ K.

Airbreathing Hypersonic Engine with MHD Energy Management System

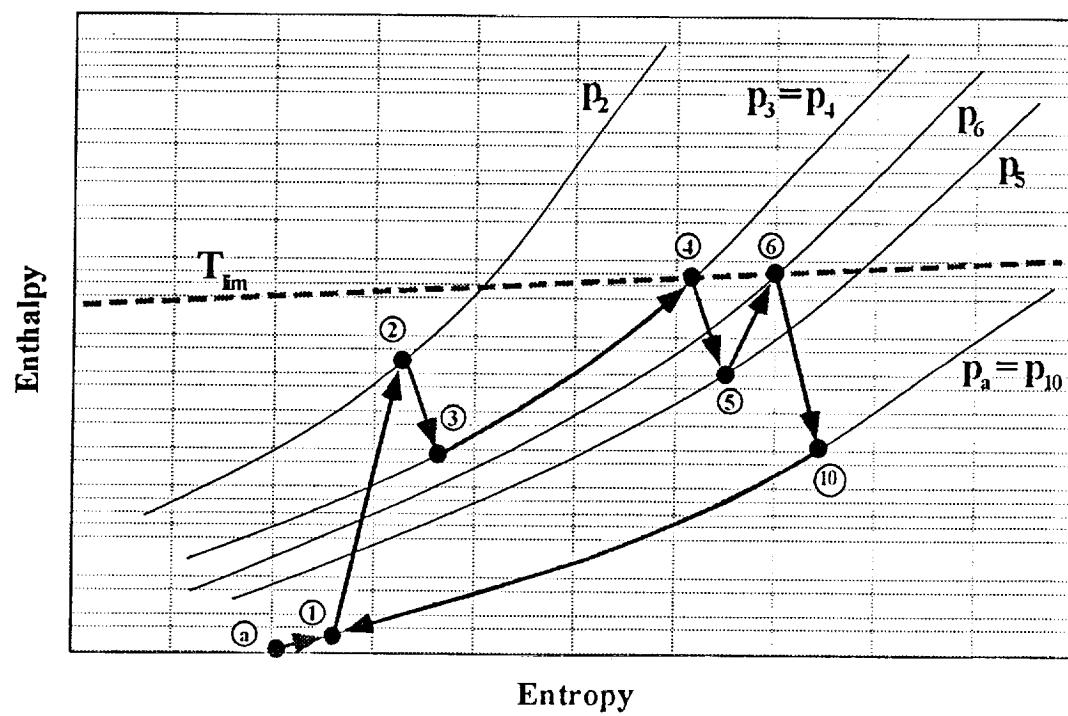
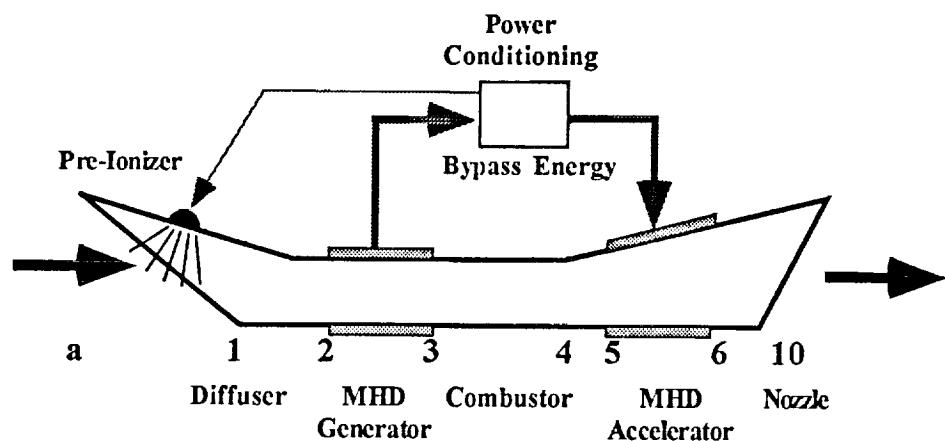


Figure 1

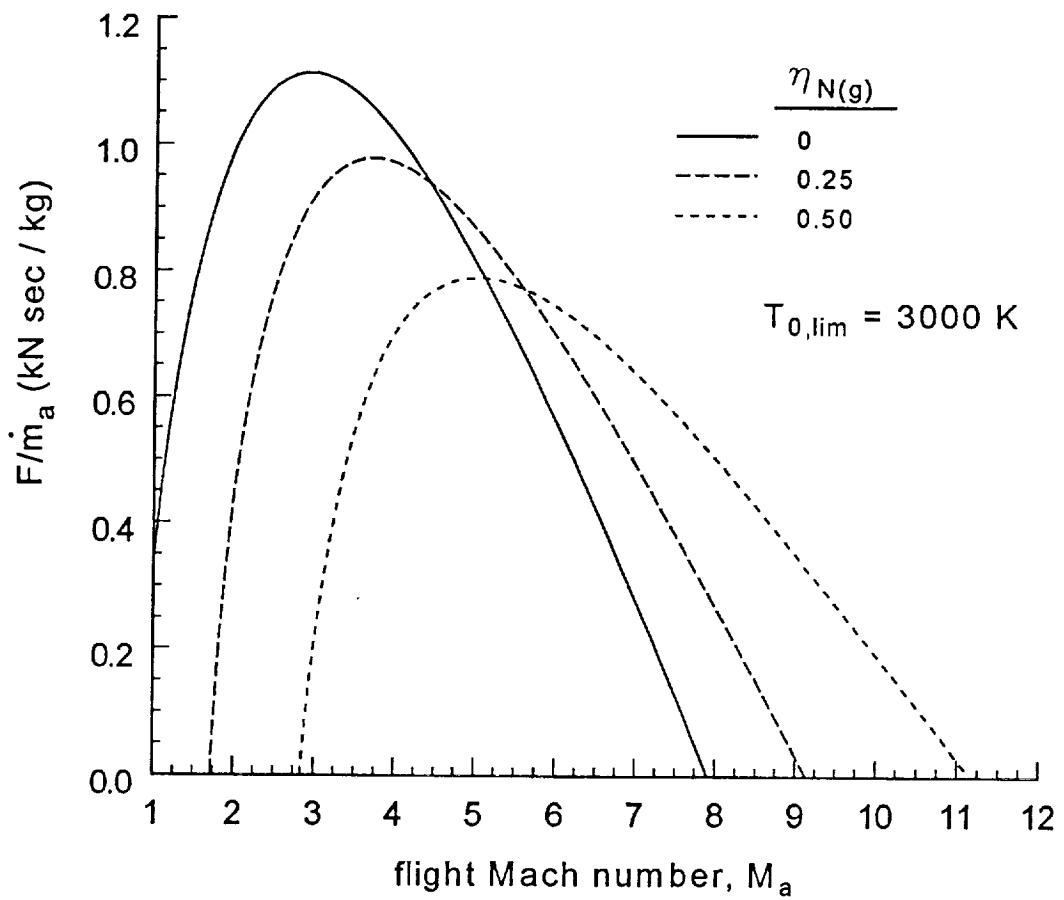


Figure 2

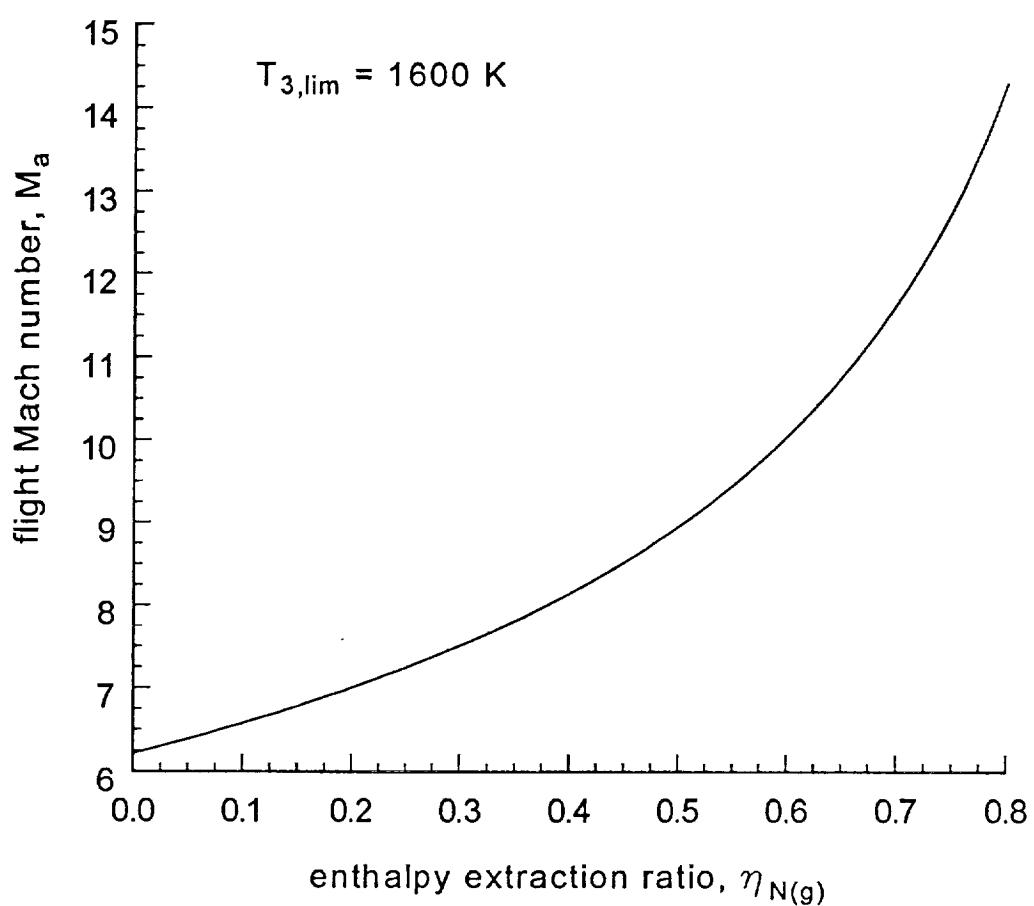


Figure 3